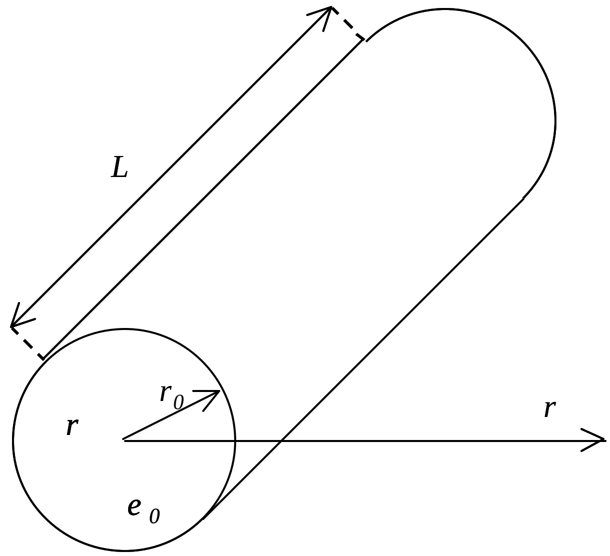
Example 1.4 Consider an infinitely long cylinder with charge density r, dielectric constant e_0 and radius r_0. What is the electric field in and around the cylinder?



Solution Because of the cylinder symmetry one expects the electric field to be only dependent on the radius, r. Applying Gauss's law one finds:

$$E \cdot A = E 2p\ rL = \frac{Q}{e_0} = \frac{r p\ r^2 L}{e_0} \text{ for } r < r_0$$

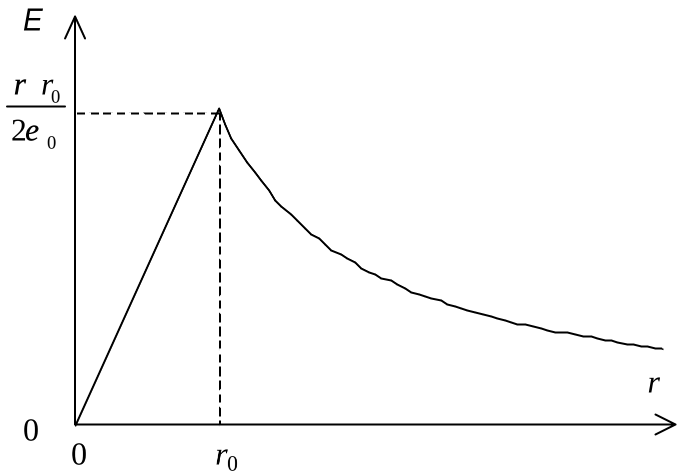
and

$$E \cdot A = E 2p\ rL = \frac{Q}{e_0} = \frac{r p\ r_0^2 L}{e_0} \text{ for } r > r_0$$

where a cylinder with length L was chosen to define the surface A, and edge effects were ignored. The electric field then equals:

$$E(r) = \frac{r\ r}{2e_0} \text{ for } r < r_0 \text{ and } E(r) = \frac{r\ r_0^2}{2e_0 r} \text{ for } r > r_0$$

The electric field therefore increases within the cylinder with increasing radius as shown in the figure below. The electric field decreases outside the cylinder with increasing radius.



E
r r0
2e 0
0
0
r0
r